University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SPECIMEN EXAMINATION (2) 2015/2016

MATH00030

Access to Science, Engineering and Agriculture: Mathematics 1

Professor G. McGuire

Dr. Anthony Cronin

Dr. Anthony Brown*

Time Allowed: 2 hours

Instructions for Candidates

Candidates should attempt all questions.
Note that not all questions are allocated the same number of marks.

Notes for Invigilators

Non programmable calculators are permitted.
The formula sheet provided is permitted.

1. (a) Without using a calculator, calculate the following.

Note that you should show enough of your working to demonstrate that you have not simply entered the expression into a calculator.

(i) $\dfrac{4}{7} - \dfrac{2}{9}$

(ii) $-\dfrac{2}{5} \times \left(-\dfrac{7}{3}\right)$

(iii) $\dfrac{2}{7} \div \dfrac{8}{5}$

(iv) $(-7)^2$

(v) $\left(\dfrac{1}{81}\right)^{-\frac{3}{4}}$

(vi) $6 \div (7 \times (-8) - 9)$

(vii) $\log_3 243$

(viii) $\log_{49} 7$ [8]

(b) Simplify the following expressions by expressing them as a power of x and/or y, as appropriate.

(i) $x^7 \times x^{-8}$

(ii) $x^{-\frac{1}{3}} \div x^{-\frac{3}{5}}$

(iii) $(x^5)^{-4}$ [3]

(c) Express $\log_a\left(\left(\dfrac{y^4}{x^2}\right)^{-3}\right)$ in terms of $\log_a x$ and $\log_a y$ [2]

(d) (i) Approximate 12.94999 to one decimal place.

(ii) Approximate 0.0001254 to two significant figures.

(iii) Express 184627.21 in scientific notation.

(iv) Express 0.0000045 in scientific notation to one significant figure.   [4]

(e) Simplify $(-2x^2 - x - 4) - (-3x - 2)$.   [1]

(f) Multiply out $(2x^4 + 3x^2)(2x - 1)$.   [2]

(g) Perform long division on $\dfrac{x^2 + 4x + 2}{x + 3}$, giving the quotient and remainder.   [4]

(h) Evaluate $\displaystyle\sum_{i=-2}^{2} -i^2$ [2]

(i) Calculate $\dbinom{10}{2}$ without using a calculator.

Note that you should show enough of your working to demonstrate that you have not simply entered the expression into a calculator.   [2]

(j) Expand $(3x + 2y)^3$ using The Binomial Theorem.   [4]

2. (a) Sketch the graph of the line with equation $y = x - 2$ concentrating on the region between $x = -2$ and $x = 6$. **[2]**

(b) Solve the simultaneous equations

$$-2x - 4y = -8$$
$$4x - 2y = -4$$

[3]

(c) Find the length of the line segment between $(-1, -2)$ and $(2, 3)$ **[1]**

3. (a) Write the expression $4x^2 - 5x + 1$ in completed square form. **[3]**

(b) Solve the equation $4x^2 - 5x + 1 = 0$ by using the quadratic formula. **[2]**

(c) Sketch the graph of the function $y = 4x^2 - 5x + 1$, showing the y-intercept, the x-intercept(s) (if applicable) and the turning point. **[4]**

4. (a) For each of the following:

- Say whether or not it is a function and if not say why not.
- If it is a function state the domain and the codomain.

(i)

$$f \colon \mathbb{R}^- \to \mathbb{R}^-$$
$$x \mapsto -x - 2$$

(ii)

$$f \colon \mathbb{R}^- \to \mathbb{R}^+$$
$$x \mapsto x^2 + 2$$

[4]

(b) Sketch the graph of the function

$$f \colon \{-3, -1, 0, 1, 3\} \to \{-3, -1, 0, 1, 3\}$$
$$-3 \mapsto 3$$
$$-1 \mapsto 1$$
$$0 \mapsto 0$$
$$1 \mapsto -1$$
$$3 \mapsto 3$$

[2]

(c) Figure 1 contains the graphs of four of the following functions:

   (i) $y = 3^x$

   (ii) $y = -2^x$

   (iii) $y = -\left(\dfrac{2}{7}\right)^x$

   (iv) $y = \log_e(x)$

   (v) $y = \left(\dfrac{7}{8}\right)^x$

   (vi) $y = \log_{1/5}(x)$
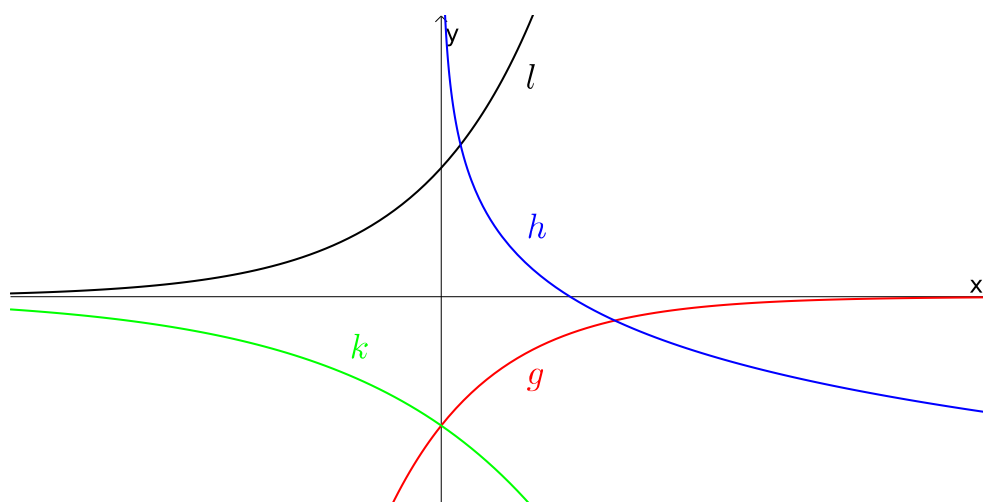
   Match the functions to the graphs. [4]

Figure 1: The functions for Question 4 (c).

(d) For each of the following functions, say whether they are injective, surjective or bijective. If a function is not injective or surjective then say why not.

   (i)

$$f\colon \{1, 2, 3, 4\} \to \{A, B, C, D\}$$
$$1 \mapsto A$$
$$2 \mapsto D$$
$$3 \mapsto B$$
$$4 \mapsto B$$

   (ii)

$$f\colon \mathbb{R}^- \to \mathbb{R}^-$$
$$x \mapsto 2x$$

[3]

(e) State whether each of the functions in Part (d) has an inverse function or not, giving a reason in each case. **[1]**

5. (a) Convert $105°$ to radians, leaving your answer as a multiple of π. **[1]**

(b) Convert $\dfrac{7\pi}{8}$ radians to degrees. **[1]**

(c) Using the geometric method, find $\tan\left(-\dfrac{2\pi}{3}\right)$ without using a calculator. **[4]**

(d) Using whichever trigonometric formulae you like, but without using a calculator, calculate the following.
Note that you should show enough of your working to demonstrate that you have not simply entered the expression into a calculator.

(i) $\sin\left(\dfrac{3\pi}{4}\right)$

(ii) $\tan\left(-\dfrac{\pi}{12}\right)$ **[4]**

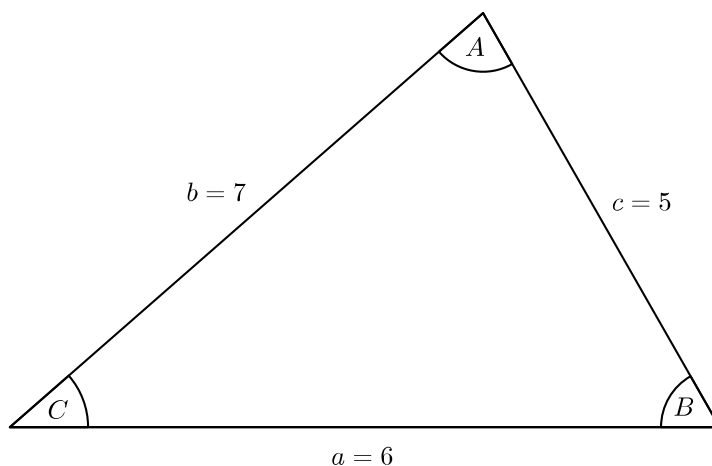(e) Find the size of the angle B in the triangle in Figure 2. **[3]**

Figure 2: The triangle for Question 5 (e).

6. (a) Find the derivative of $f(x) = -x^2$ using first principles. **[3]**

(b) Find the derivatives of the following functions.

(i) $f(x) = e^{\cos(2)} + 1$

(ii) $f(x) = x^4$

(iii) $f(x) = \cos(-4x)$

(iv) $f(x) = \sin(2x)$

(v) $f(x) = -3x^{-\frac{1}{3}} - 3e^{-3x} - 3\ln(-3x)$ (where $x < 0$) **[6]**

7. Find the following integrals.

(a) $\displaystyle\int 0 \, dx$ **[1]**

(b) $\displaystyle\int_{-1}^{1} x^4 \, dx$ **[2]**

(c) $\displaystyle\int_{0}^{\frac{\pi}{2}} \sin(3x) \, dx$ **[2]**

(d) $\displaystyle\int e^{-x} - 3x^{-\frac{4}{5}} \, dx$ **[2]**

8. (a) For the list of numbers $1, 3, 3, -6, 5, 6, 1, 2$, find the

    (i) Mean

    (ii) Median

    (iii) Mode(s)

    (iv) Interquartile range **[5]**

(b) Find the line of best fit using the least squares method with the points $(-4, 3), (-2, 1), (0, 1), (3, -1)$ and $(5, -4)$. **[8]**

—o0o—